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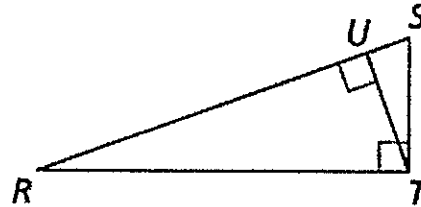
Similar Right Triangles

Lesson Objective **INBAT** identify and use ~ right Δ s and use geometric means.

When the altitude is drawn to the hypotenuse of a right triangle, the two smaller triangles are similar to the original triangle and to each other.

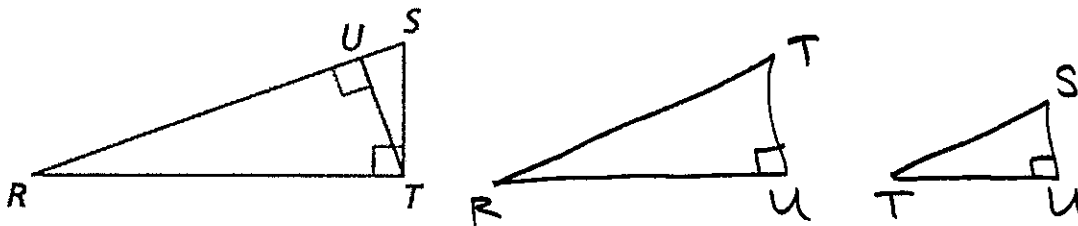
Right Triangle Similarity Theorem

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



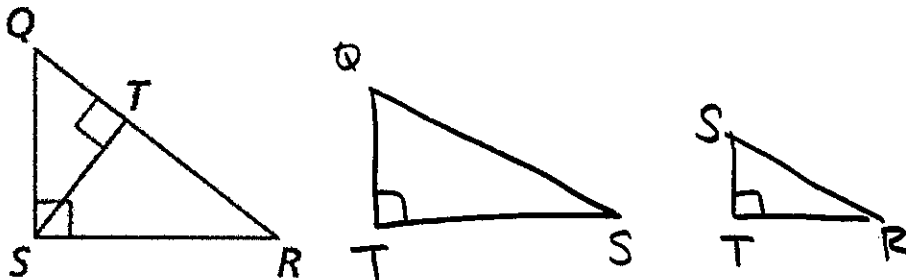
Example:

1. Identify the similar triangles in the diagram, and write a similarity statement.



$$\Delta RST \sim \Delta RTU \sim \Delta TSU$$

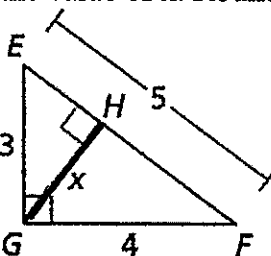
2. Identify the similar triangles in the diagram, and write a similarity statement.



$$\Delta QSR \sim \Delta QTS \sim \Delta STR$$

3. Find the value of x . Round to the nearest hundredth, if necessary.

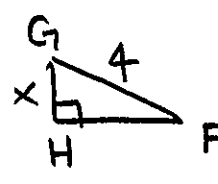
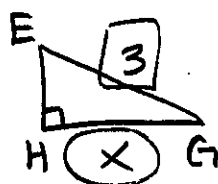
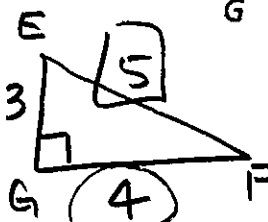
a.



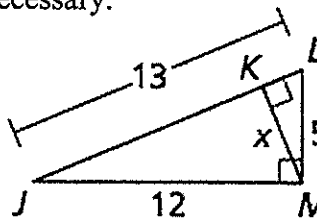
$$\frac{5}{3} = \frac{4}{x}$$

$$5x = 12$$

$$x = 2.4$$



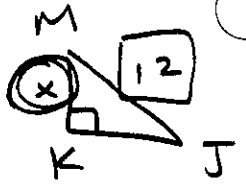
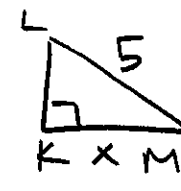
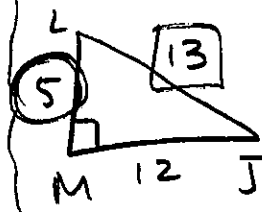
b.



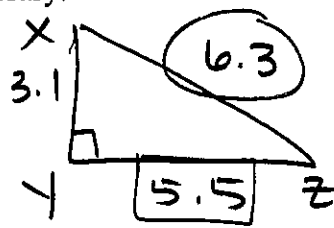
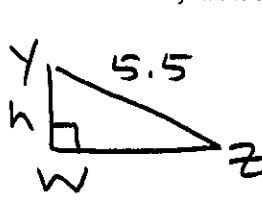
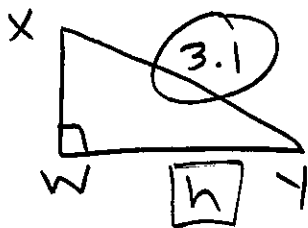
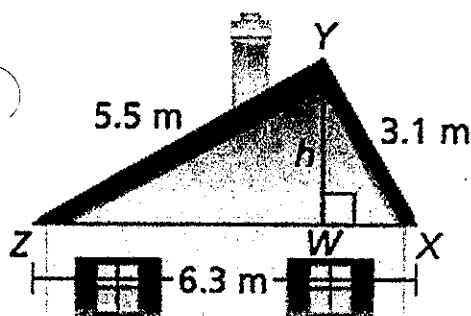
$$\frac{x}{5} = \frac{12}{13}$$

$$13x = 60$$

$$x \approx 4.62$$



4. A roof has a cross section that is a right triangle. The diagram shows the approximate dimensions of this cross section. Find the height h of the roof. Round to the nearest hundredth, if necessary.



$$\frac{h}{5.5} = \frac{3.1}{6.3}$$

$$h \approx 2.71 \text{ m}$$

Geometric Mean: The geometric mean of two positive integers a and b is the positive number x that satisfies

$$\frac{a}{x} = \frac{x}{b}$$

Example:

5. Find the geometric mean of 24 and 48. Find the EXACT value, and round to the nearest hundredth, if necessary.

~~74.4~~

$$\frac{24}{x} = \frac{x}{48}$$

$$x^2 = (24)(48)$$

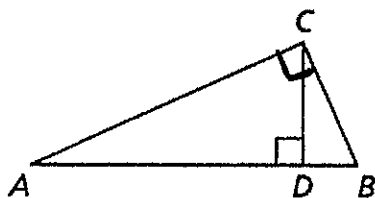
$$x = \sqrt{4 \cdot 6 \cdot 4 \cdot 12}$$

$$x = 2 \cdot 2 \sqrt{2 \cdot 3 \cdot 3 \cdot 4}$$

$$x = 4 \cdot 2 \cdot 3 \sqrt{2} \rightarrow x = 24\sqrt{2}$$

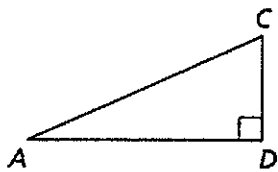
$$x \approx 33.94$$

In right $\triangle ABC$, altitude $\overline{CD}$ is drawn to the hypotenuse, forming two smaller right triangles that are similar to $\triangle ABC$. From the Right Triangle Similarity Theorem, you know that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$. Because the triangles are similar, you can write and simplify the following proportions involving geometric means.



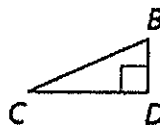
$$\frac{AD}{CD} = \frac{CD}{DB}$$

$$CD^2 = (AD)(DB)$$



$$\frac{AD}{AC} = \frac{AC}{AB}$$

$$AC^2 = (AD)(AB)$$



$$\frac{BD}{BC} = \frac{BC}{AB}$$

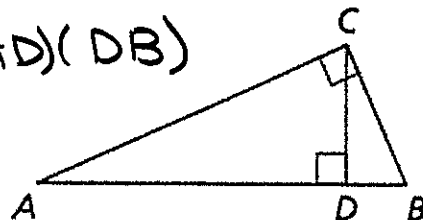
$$BC^2 = (BD)(AB)$$

Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments of the hypotenuse.

$$CD^2 = (AD)(DB)$$



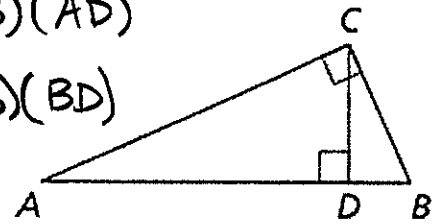
Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

$$AC^2 = (AB)(AD)$$

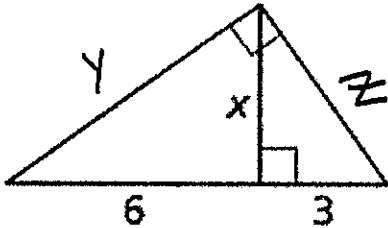
$$BC^2 = (AB)(BD)$$



Examples:

6. Find the value of each variable.

a.



$$z^2 = (9)(3)$$

$$z = \sqrt{9 \cdot 3}$$

$$z = 3\sqrt{3}$$

$$x^2 = (6)(3)$$

$$x = \sqrt{18}$$

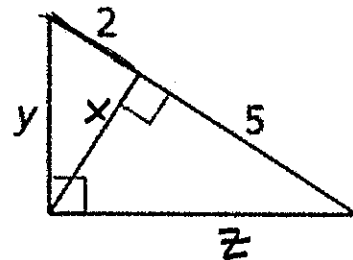
$$x = 3\sqrt{2}$$

$$y^2 = (9)(6)$$

$$y = \sqrt{9 \cdot 6}$$

$$y = 3\sqrt{6}$$

b.



$$x^2 = (2)(5)$$

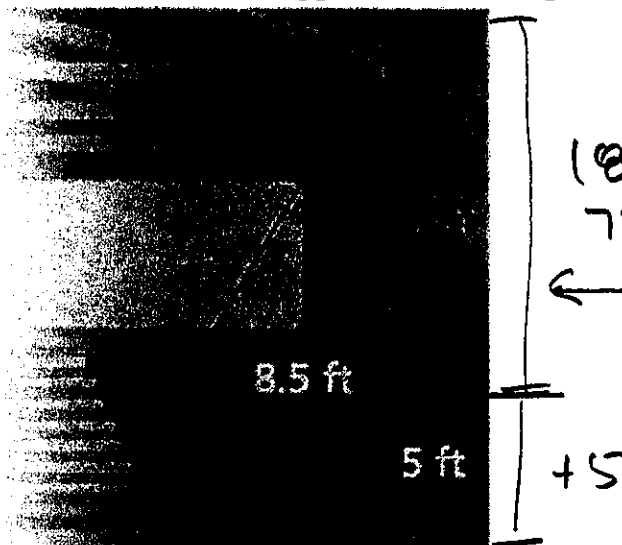
$$x = \sqrt{10}$$

$$y^2 = (7)(2)$$

$$y = \sqrt{14}$$

$$z^2 = (7)(5) \Rightarrow \sqrt{35} = z$$

7. To find the cost of installing a rock wall in your school gymnasium, you need to find the height of the gym wall. You use a cardboard square to line up the top and bottom of the gym wall. Your friend measures the vertical distance from the ground to your eye (5 ft) and the horizontal distance from you to the gym wall (8.5 ft). Approximate the height of the gym wall. Round to the nearest hundredth.



$$(8.5)^2 = (5)(w)$$

$$72.25 = 5w$$

$$w = 14.45 \text{ ft}$$

$$\text{Wall is } 14.45 + 5 = 19.45 \text{ ft tall}$$